

# Probability and statistics midterm exam (Model Three)

## A-Probability

### Question (1) (10 points):

A-Determine the probability  $p$ , or an estimate of it, for each of the following events:

- (a) A king, ace, jack of clubs or queen of diamonds appears in drawing a single card from a well- shuffled ordinary deck of cards.
- (b) The sum 8 appears in a single toss of a pair of fair dice.
- (c) A nondefective bolt will be found next if out of 600 bolts already examined, 12 were defective.
- (d) At least one head appears in three tosses of a fair coin.

B-Prove that if  $P(A) > P(B)$  then  $P(A | B) > P(B | A)$ .

C-Urn I has 2 white and 3 black balls; Urn II, 4 white and 1 black; and Urn III, 3 white and 4 black.

An urn is selected at random and a ball drawn at random is found to be white. Find the probability that Urn I was selected.

### Question (2) (10 points):

A- An urn holds 5 white and 3 black marbles. If two marbles are drawn at random without replacement and  $X$  denotes the number of white marbles, (a) find the probability distribution for  $X$  and (b) graph the distribution.

B- Let  $X$  be a random variable having density function  $f(x)$

Find (a) the value of the constant  $c$ , (b)  $P(1/2 < X < 3/2)$ , (c)  $P(X > 1)$ ,

(d) the distribution function.

$$f(x) = \begin{cases} cx & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

### Question (3) (10 points):

A- Find the probability of guessing correctly at least 6 of the 10 answers on a true-false examination.

B- A machine produces bolts which are 10% defective. Find the probability that in a random sample of 400 bolts produced by this machine (a) at most 30, (b) between 30 and 50, (c) between 35 and 45 of the bolts will be defective.

C- Find  $z_1$ , if  $P(Z \geq z_1) = 0.84$ , where  $Z$  is normally distributed with mean 0 and variance 1.

## B- Statistics

### Question (4):

The weights of 1500 ball bearings are normally distributed with a mean of 22.40 ounces and a standard deviation of 0.048 ounces. If 300 random samples of size 36 are drawn from this population.

(a) Determine the expected mean and standard deviation of the sampling distribution of means if sampling is done (i) with replacement, (ii) without replacement.

(b) how many of the random samples would have their means between 22.39 and 22.41

### Question (5):

Measurements of a sample of weights were determined as 8.3, 10.6, 9.7, 8.8, 10.2 and 9.4 pounds respectively.

Determine unbiased and efficient estimates of (a) the population mean and (b) the population variance

(c) Compare the sample standard deviation with the estimated population standard deviation.